On the Impact of a Fundamental Trade-off between Costs and Precision on the Fate of Dynamical Systems

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In biological systems, information transfer is realized in the transcription and translation machinery, in molecular motors or pumps and enzymatic reactions. For these biomolecular processes a thermodynamic uncertainty relation has been derived. It states that a more precise output (such as the number of product molecules generated by an enzymatic reaction) requires a higher thermodynamic cost independent of the time used to produce the output. Applied to molecular motors which provide an example for a driven system that moves with time t along a coordinate x, the uncertainty relation implies an upper bound on its thermodynamic efficiency: the higher the precision, that is, the smaller the dispersion or uncertainty in x, the higher the entropy production, and the smaller the efficiency. Another example for information transfer and transformation is realized in cellular sensing. Living cells measure low chemical concentrations with high but limited precision. The fundamental sensing limit results from different resource classes, one of which is energy. In particular it has been shown why cellular sensing systems can never reach the Landauer limit on the optimal trade-off between accuracy and energetic costs. In view of all these inherent error-prone processes, nature has invented correction mechanisms like kinetic proofreading, that is a mechanism for error correction in biochemical reactions. However, also there, in conformational proofreading, there is a trade-off between product production and its efficiency. Theoretically the proofreading can achieve infinite specificity the longer the reaction runs, but at the costs of large amounts of the (correct) product as well.

In this contribution we abstract from the concrete realizations and consider a minimalist model for analyzing the influence of a fundamental trade-off between costs and precision on the dynamical performance of a system. An essential characteristics of the dynamics is a cyclic process that should run at low entropy, here chosen to be an error-prone copying process of an ordered bitstring. Without repair, the bitstring gets either randomized, going along with a complete loss of the original structure, or the finite accessible energy reservoir gets depleted before the string is randomized. When the copying process is combined with a repair process with constant probability of the success of repair, it is possible to sustain the cyclic performance without exceeding a desired tolerable error threshold. This is possible in spite of the fact that energy for repair is taken from the same finite reservoir from which the copying process is sustained, so repair is performed on the price of the subsequent copying accuracy. The conditions for this to happen are derived from the bifurcation diagram of a discrete map that describes the time evolution of errors. When these conditions are violated, the fate of the system is a lack of energy supply for its essential performance, since all energy gets absorbed by repair. This fate seems unavoidable when also repair succumbs the trade-off and gets less effective in the course of time, unless energy would be invested in the repair of repair. We truncate the iteration at the point where the costs for repair of repair should be taken into account in view of a complete description. Our conjecture is that a decaying success of repair over time amounts to an effective description of such an iterated procedure.